Lesson 2-3: Deductive Reasoning

The history of geometry

Do you remember what geometry means? It comes from the Greek and means “earth measurement.” Over thousands of years ago, the Babylonians and Egyptians discovered and used many geometric principles. They used these “rule of thumb” procedures for building and construction.

By the early 600’s B.C. the Greek civilization was growing and becoming more prosperous. With that prosperity came free time for discussing issues of thought, debate and study. This led to an insistence on using reasoning as the basis for statements during debate. Mathematicians then began to use logical reasoning to deduce and support mathematical ideas.

Prior to the Greeks, inductive reasoning ruled in geometry. Linking together chains of logical reasoning, the Greeks applied deductive reasoning to geometry, systematically developing what we now know as Euclidean geometry.

From ‘Discovering Geometry’ by Michael Serra, p. 718.

Now thus far, we have been using primarily inductive reasoning as we’ve explored the building blocks of Euclidean plane geometry. We have, these past few days, looked at building blocks of logic. We now put those together and begin working with deductive reasoning. But first, a story…

Good old Sherlock Holmes

One day as Sherlock Holmes was working in his lab, in walked his faithful assistant Watson. “So, Watson” said Sherlock, “you do not propose to invest in South American securities?”

As accustomed as Watson was to Sherlock’s curious faculties, this sudden intrusion into his most intimate thoughts was utterly inexplicable.

“Holmes, how on earth can you possibly know that?”

“Now Watson, confess yourself utter taken aback.”

“I am!”

“I ought to have you sign a paper to that effect.”

“Why???”

“Because in five minutes you will say that it is all absurdly simple.”

“Holmes, I am sure that I shall say nothing of the kind!”

“You see my dear Watson, it is not really difficult to construct a series of inferences, each dependent upon its predecessor and each simple in itself. If, after doing so, one simply knocks out all the central inferences and presents one’s audience with the starting point and the conclusion, one may produce a startling, though possibly a meretricious, effect. Now, it is not really difficult, by inspection of the groove between your left forefinger and thumb, to feel sure that you did not propose to invest your small capital in the gold fields."

"I see no connection."

"Very likely not; but I can quickly show you a connection. Here are the missing links of the very simple chain:"

1) You had chalk between your left finger and thumb when you returned from the club last night.

2) You put chalk there when you play billiards, to steady the cue.

3) You never play billiards except with Thurston.

4) You told me four weeks ago that Thurston had an option on some South African property that would expire in a month, and which he desired you to share with him.

5) Your checkbook is locked in my drawer, and you have not asked for the key.

6) You do not propose to invest your money in this manner."

"How absurdly simple" cried Watson.

Adapted from 'The Adventure of the Dancing Men' by Sir Arthur Conan Doyle.

What form of reasoning did Sherlock use? He built a chain of inferences or deductions, each of which came from the prior. Once he got to the end of the chain, he simply removed all links and put the first and the last together in a single statement.

This is deductive reasoning. **Most simply put, deductive reasoning is reasoning from the general to the specific.** This is in contrast to inductive reasoning which is reasoning from the specific to the general. If a given statement is true, deductive reasoning produces a true conclusion.

Two laws of deductive reasoning

There are two laws of deductive reasoning that we need to be aware of. They have strange names and may sound confusing but we will do our best to make them clear. The first is the Law of Detachment and the second is the Law of Syllogism.

The Law of Detachment

The Law of Detachment states:

> If a conditional is true and its hypothesis is true, then its conclusion is true.
> In symbolic form: if $p \rightarrow q$ is a true statement and p is true, then q is true.

Hmm, not a lot of words, no new or confusing words, but what does it mean???

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The simplest way I can put it is this:

> If you know a conditional statement to be true in general, and you find a specific situation for which the hypothesis is true, you can then be positive the conclusion applies to this situation.

The basic steps are:
1. Determine if the conditional is true. Stop if not.
2. Determine if the situation relates directly to the conditional's hypothesis. Stop if not.
3. Then the conclusion applies and is true.

Still confused? Consider the Sherlock Holmes story above. Holmes used this law implicitly.

- He had a general conditional he knew to be true: if a person returns from the billiards club with chalk on their offhand forefinger and thumb, then they have been playing billiards.
- He encountered a situation in which he noted that Watson had been at the billiards club and had chalk on his finger and thumb. The general hypothesis applied.
- Given that the hypothesis applied in this situation, he knew the conclusion also applied.
- Thus by examining the groove between Watson's finger and thumb, Sherlock concluded that Watson had been playing billiards.

Let's try another example. Look at *Check Understanding* 2 on page 83. If possible, use the Law of Detachment to draw a conclusion. If it is not possible to use this law, explain why.

> **Given**: If a baseball player is a pitcher, then that player should not pitch a complete game two days in a row. (General conditional)
> Vladimir Nuňez is a pitcher. On Monday, he pitches a complete game. (Specific situation)

What do you think? Does the Law of Detachment apply here? We have a general conditional. We have a specific situation. Does the hypothesis of the general conditional directly apply to the situation? **Yes it does. We can use the Law of Detachment in this situation. Vladimir should not pitch a complete game on Tuesday.**

Let's try another one. Look at *Check Understanding* 3 on page 83.

> **Given**: If a road is icy, then driving conditions are hazardous. (General conditional)
> Driving conditions are hazardous. (Specific situation)

Does the hypothesis of the general conditional directly apply to the situation? **No it doesn't; the situation is related to the conclusion, not the hypothesis. We can not use the Law of Detachment in this situation.**

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The Law of Syllogism

This next law of deductive reasoning Sherlock used explicitly when he built the chain of conclusions. The Law of Syllogism states (in symbolic form):

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.

This is much easier to understand. It is exactly what Sherlock did. If you can build a chain of conditional statements where the conclusion if each is the hypothesis of the following, then you can assume the conclusion of the last follows directly from the hypothesis of the first. Thus you can say:

If $p \rightarrow q$ and $q \rightarrow r$ and $r \rightarrow s$ and $s \rightarrow t$ then $p \rightarrow t$

This is a **very** powerful tool. It is perhaps the main logic tool we will be using as we work with geometric proofs.

Looking at the above chain, does it remind you of anything? It is symmetric with the *Transitive Property of Addition* which states:

If $a = b$ and $b = c$, then $a = c$.

Same basic idea…

Example – p. 84, Check Understanding #4

If possible, state a conclusion using the Law of Syllogism. If it is not possible to use this law, explain why.

a) If **a number ends in 0**, then **it is divisible by 10**. *($p \rightarrow q$)*
If **a number is divisible by 10**, then **it is divisible by 5**. *($q \rightarrow r$)*

Here we have a clear and consistent chain of conditionals: $p \rightarrow q$ and $q \rightarrow r$
Both conditionals are true.
Therefore we can conclude $p \rightarrow r$.
If a number ends in 0, then it is divisible by 5.

b) If **a number ends in 6,** then **it is divisible by 2**. *($p \rightarrow q$)*
If **a number ends in 4,** then **it is divisible by 2**. *($r \rightarrow q$)*

This one is different:
The conclusion of one statement is not the hypothesis of the other.
Therefore it is not possible to apply the Law of Syllogism.

Assign homework

p. 84 1-15, 23-31 odd, 38-44
p. 88 1-10